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John S. Malik

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87545

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ABSTRACT

A method of inverting the geomagnetic component of the radioflash signal from a nuclear explosion to obtain the gamma-ray time history was proposed by E. D. Dracott of the Atomic Weapons Research Establishment. A simplified development of an elaboration by B. R. Suydam has been programmed for small calculators in a form suitable for interim field analysis of such data. The development of the program is contained in the report.

I. INTRODUCTION

The most useful component of the radioflash signal emitted during a nuclear weapon explosion for diagnostic purposes is the geomagnetic component. As its name implies it results from the deflection of the Compton electrons, produced by interaction of gamma rays with air molecules, by the static geomagnetic field of the earth. It has the character of a magnetic dipole signal; the radiated electric field is perpendicular to the geomagnetic field. Its generation has been described in several reports, notably by Suydam.[1] Its potential for the determination of the gamma time history of the source producing the signal was recognized first by E. D. Dracott.[2] The implementation of the proposal has been by Moody and Hill[3] with further extension by Suydam.[4] Suydam's method has been implemented by Malik[5] in the Los Alamos Scientific Laboratory (LASL) program HMSD. Program GRUF is a simplified unfolding technique which has been programmed for the Wang 700 and Hewlett-Packard 9820 calculators; it is suitable for early data reduction while in the field. As in the development of the program HMSD,[5] the development of the program GRUF closely follows Suydam's.[4]

II. DEVELOPMENT

The starting point is Maxwell's equations in MKS units:

$$\frac{\partial(\epsilon E)}{\partial t} + J_{Tot} = \text{curl } (B/\mu) \qquad (1)$$

$$\frac{\partial B}{\partial t} = - \text{curl } E \quad . \qquad (2)$$

In polar coordinates, the magnetic or TE set of equations is

$$\epsilon \frac{\partial E_\phi}{\partial t} + \sigma E_\phi = - J_\phi + \frac{1}{\mu r} \frac{\partial}{\partial r} (rB_\theta) - \frac{1}{\mu r} \frac{\partial B_r}{\partial \theta} \qquad (3)$$

$$\frac{\partial B_r}{\partial t} = - \frac{1}{r} \sin\theta \frac{\partial}{\partial \theta} (\sin\theta \; E_\phi) \qquad (4)$$

$$\frac{\partial B_\theta}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (rE_\phi) \quad . \qquad (5)$$

Transforming to retarded time coordinates: $r' = r$, $\tau = t - r/c$, for which the operators become $\frac{\partial}{\partial r} \to \frac{\partial}{\partial r'} - \frac{1}{c} \frac{\partial}{\partial \tau}$, $\frac{\partial}{\partial t} \to \frac{\partial}{\partial \tau}$ and dropping the prime on r, the B_θ equation becomes:

$$\frac{\partial B_\theta}{\partial \tau} = \frac{1}{r}\frac{\partial (rE_\phi)}{\partial r} - \frac{1}{c}\frac{\partial E_\phi}{\partial \tau} \quad . \tag{6}$$

The B_θ and B_r Eqs. (4) and (5) are integrable to

$$B_\theta = -\frac{1}{c}E_\phi + \frac{1}{r}\int \frac{\partial (rE_\phi)}{\partial r}\,d\tau \tag{7}$$

$$B_r = -\frac{1}{r}\sin\theta \int \frac{\partial(\sin\theta\, E_\phi)}{\partial\theta}\,d\tau \ . \tag{8}$$

In the transformed coordinates, Eq. (3) becomes

$$c\frac{\partial E_\phi}{\partial \tau} + \sigma E_\phi = -J_\phi + \frac{1}{\mu r}\frac{\partial (rB_\theta)}{\partial r} - \frac{1}{\mu r}\frac{\partial B_r}{\partial \theta} - \frac{1}{\mu c}\frac{\partial B_\theta}{\partial \tau} \ . \tag{9}$$

Substituting for B_r and B_θ in Eq. (9),

$$\sigma E_\phi = -J_\phi - \frac{1}{\mu c r}\frac{\partial (rE_\phi)}{\partial r} + \frac{1}{\mu r}\int \frac{\partial^2 (rE_\phi)}{\partial r^2}\,d\tau$$

$$+ \frac{1}{\mu}\sin\theta \ \frac{1}{r^2}\int \frac{\partial^2(\sin\theta\, E_\phi)}{\partial\theta^2}\,d\tau \ . \tag{10}$$

By restricting the solutions to short times, the terms involving integrals may be dropped:

$$\frac{d(rE_\phi)}{dr} + \frac{1}{2}\sqrt{\frac{\mu}{\varepsilon}}\,\sigma(rE_\phi) = -\frac{1}{2}\sqrt{\frac{\mu}{\varepsilon}}\,(r\,J_\phi) . \tag{11}$$

This is the starting point for Suydam's development.

Choosing the polar axis antiparallel to the earth's magnetic field, the transverse current J_ϕ at the azimuthal angle θ is given by

$$J_\phi = \frac{\ell}{2a}J_c \sin\theta \tag{12}$$

where ℓ is the range of the Compton electron, a the Larmor radius, J_c the Compton current. Equation (11) integrates to

$$(rE_\phi) = \frac{\ell}{2a}\sin\theta \quad e^{\chi}\int_0^r rE_s\, e^{-\chi}\frac{Z_0}{2}\sigma\,dr \tag{13}$$

with $E_s \equiv -J_c/\sigma$ (14)

$$\chi = \int_r^\infty \frac{Z_0}{2}\sigma\,dr \tag{15}$$

ℓ = electron range
a = Larmor radius
$Z_0 = \sqrt{\mu/\varepsilon}$.

At large distances ($\sigma \to 0$), $\chi \to 0$ and

$$(rE_\phi) = \frac{\ell}{2a}\sin\theta\int_0^\infty rE_s\, e^{-\chi}\frac{Z_0}{2}\sigma\,dr \ . \tag{16}$$

Recognizing the azimuthal dependence, we will omit it temporarily and solve for rE_ϕ at $\theta = \pi/2$. The gamma-ray dependent functions are approximated by

$$J_c = \frac{Y}{r^2}e^{-r/\lambda_T} f(\tau) \tag{17}$$

$$\sigma = \frac{\Sigma}{r^2}e^{-r/\lambda_T} g(\tau)$$

with

$$Y = \frac{e\ell}{4\pi\lambda_c W_\gamma} \qquad \Sigma = \frac{e\mu_e}{4\pi\lambda_a W_{ip}} \tag{18}$$

where μ_e is the electron mobility, W_{ip} is energy per ion pair, e the electron charge, and λ_c, λ_a are γ-ray Compton and absorption mean free paths. The function g has dimensions of energy; f, that of energy/time. They are connected by the air-ion equation:

$$\frac{\partial g}{\partial \tau} + \beta g = f \tag{19}$$

where β is the electron attachment rate to

O_2: it is the only important loss rate for the times of interest. The electron mobility μ_e is related to conductivity by $\sigma = ne\mu_e$ with n being the electron density.

Denoting time derivatives by dot, E_s as defined by Eq. (14) becomes

$$E_s = [illegible] = \frac{Y}{\Sigma}\left(\beta + \frac{\dot g}{g}\right) \quad . \tag{20}$$

Writing the equation for rE_ϕ, Eq. (16) as

$$rE_\phi = \frac{\ell}{2a} R_s E_s (1 + Q_m) \tag{21}$$

where R_s is the value of r satisfying

$$\frac{Z_0 \sigma}{2} = -\frac{1}{J}\frac{\partial J}{\partial r} \quad . \tag{22}$$

The correction quantity Q_m is tabulated by Suydam.[4] Equations (21) and (22) are general. Specializing J and σ to Eqs. (17), Eq. (22) becomes

$$g(\tau) = \frac{2\lambda_T}{Z_0 \Sigma} x(x+2)\, e^x \tag{23}$$

where

$$x = R_s/\lambda_T \tag{24}$$

and λ_T is the γ-ray transport mean free path. By logarithmic differentiation:

$$\frac{\dot g}{g} = \left(1 + \frac{2(x+1)}{x(x+2)}\right)\dot x \tag{25}$$

whence

$$E_s = \frac{Y}{\Sigma}\left\{\beta + \dot x\left[1 + \frac{2(x+1)}{x(x+2)}\right]\right\}. \tag{26}$$

The ratio Y/Σ is given by

$$\frac{Y}{\Sigma} = \frac{\dfrac{e\ell}{4\pi\lambda_c W_\gamma}}{\dfrac{\nu\mu_e}{4\pi\lambda_a W_{ip}}} = \frac{\ell W_{ip}\lambda_a}{\mu_e W_\gamma \lambda_c} = \frac{\ell}{\nu\mu_e} \tag{27}$$

where ν is the number of secondary electrons produced per primary γ-ray of energy W_γ. Putting Eqs. (26) and (27) into Eq. (21),

$$rE_\phi = \frac{\ell}{2a}\cdot\frac{\ell\lambda_T}{\nu\mu_e}\, x\left\{\beta + \dot x\left[1 + \frac{2(x+1)}{x(x+2)}\right]\right\}(1+Q_m) \tag{28}$$

or

$$K_1(rE_\phi) = x\left[\beta + \dot x\,(1+\eta)\right](1+Q_m) \tag{29}$$

with

$$K_1 = \frac{2a}{\ell}\cdot\frac{\nu\mu_e}{\ell}\cdot\frac{1}{\lambda_T} = \frac{2a}{\ell}\cdot\frac{W_\gamma\lambda_c}{W_{ip}\lambda_a}\frac{\mu_e}{\ell}\frac{1}{\lambda_T} \tag{30}$$

$$\eta = \frac{2(x+1)}{x(x+2)} \quad . \tag{31}$$

Equation (29) in the form

$$\dot x = \frac{K_1(rE_\phi)/(1+Q_m) - \beta x}{x(1+\eta)} \tag{32}$$

gives the differential equation for x or R_s. Using Eq. (23) gives $g(\tau)$. Combining Eqs. (20) and (21)

$$rE_\phi = \frac{\ell}{2a}\cdot\frac{\lambda_T Y}{\Sigma}\frac{f}{g}\, x(1+Q_m) \tag{33}$$

or

$$\begin{aligned} f &= \frac{2a}{\ell}\frac{\Sigma}{Y\lambda_T}\frac{(rE_\phi)}{(1+Q_m)}\frac{g}{x} \\ &= \frac{2a}{\ell}\cdot\frac{2}{Z_0 Y}\frac{(rE_\phi)}{(1+Q_m)}(x+2)e^x \\ &= K_3\frac{(rE_\phi)}{(1+Q_m)}(x+2)e^x \end{aligned} \tag{34}$$

$$K_3 = \frac{4a}{\ell\,\dfrac{Z_0\cdot e\ell}{4\pi\lambda_c W_\gamma}} = \frac{4a\lambda_c W_\gamma}{e\ell^2}\cdot\frac{4\pi}{Z_0} \tag{35}$$

$$K_2 = K_3/K_1 = \frac{2W_{ip}\lambda_a\lambda_T}{e\mu_e}\cdot\frac{4\pi}{Z_0} \quad . \tag{36}$$

From a measurement of rE_ϕ, integration of Eq. (32) followed by a substitution into Eq. (34) yields the gamma-ray time dependence f. This assumes a knowledge of the other parameters in these relations, most of which are functions of the γ-ray energy. Since the source covers a spectrum of γ-ray energies from below 0.1 MeV to perhaps 10 MeV, a weighted average over the spectrum is needed. As the variable $x = R/\lambda_T$ is not a linear function of distance, the averaged variables were fitted as functions of x rather than the radial distance. The electron mobility is approximated by

$$\mu_e = \frac{1.67 \times 10^{-9}}{(1+2p)(\rho/\rho_0)} \ m^2/v\ ns$$

where p is the molecular per cent of water in the air. The electron attachment rate is approximated by

$$\beta = 0.09 \left(\frac{\rho}{\rho_0}\right)^2 ns^{-1} \quad .$$

The correction factor for the integral is fit by

$$Q_m = (5.07 + 4.61x)/(1+16x+8.36x^2) \quad .$$

The quantity K_1 as the result of the averaging is

$$K_1 = \frac{(0.189+0.0264x+0.0141x^2)(\rho/\rho_0)^2}{6750(1+0.573x+0.846x^2)(1+2p)}$$

$(\text{volt ns})^{-1}$

$$K_3 = \frac{(0.47 + 0.094x^2)10^{17} \ (\rho/\rho_0)}{3000(1 + 0.78x^2)}$$

$(\text{MeV volt ns})^{-1}$

The measured vertical electric field, E_V, obtained at a distance R in meters co-altitude with the source at an aximuthal angle θ and at a region where the dip angle is ψ, relates to the electric field rE_ϕ in the Eqs. (32) and (34) by

$$rE_\phi = RE_V/\ (\cos\psi \cdot \sin\theta) \quad .$$

The angle θ is the supplement of the true bearing to the source minus the magnetic declination.

The integration is performed by a second order Rünge-Kutta scheme:

$$y_{i+1} = y_i + 0.5\ (p_1+p_2) + O(h^3)$$

$$p_1 = h \cdot f(x_i, y_i)$$

$$p_2 = h \cdot f(x_i + h,\ y_i + p_1)$$

$$f = y'$$

or in this instance:

$$p_1 = \Delta t \cdot \dot{x}(V_{i-1},\ x_{i-1})$$

$$p_2 = \Delta t \cdot \dot{x}\ (V_i,\ x_{i-1} + p_1)$$

$$x_i = x_{i-1} + 0.5\ (p_1 + p_2)$$

where

$$\dot{x}(V,\ x) = \frac{K_1\ V/Q - \beta x}{(2+4x+x^2)/(x+2)} \quad .$$

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